COMPETITION AND INTEGRATION AMONG COMPLEMENTS, AND NETWORK MARKET STRUCTURE*

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This article analyzes the competition and integration among complementary products that can be combined to create composite goods or systems. The model generalizes the Cournot duopoly complements model to the case in which there are multiple brands of compatible components. It analyzes equilibrium prices for a variety of organizational and market structures that differ in their degree of competition and integration. The model applies to a variety of product networks including ATMs, real estate MLS, airlines CRS, as well as to non-network markets of compatible components such as computer CPUs and peripherals, hardware and software, and long distance and local telephone services.

I. INTRODUCTION

RECENT advances in the analysis of issues of product compatibility and networks have focused renewed attention on complementary goods. Production and distribution networks often are composed of both competing and complementary brands of components. The complementary components then can be combined to produce composite products or systems, which are substitutes for one another.

This formulation applies to complementary components such as mutually compatible hardware and software e.g. personal computers and software, VCRs and video tape, etc. Many electronic communication networks also can be analyzed in the same fashion. For example, an Automatic Teller Machine (ATM) network is composed of ATMs and bankcards. The consumer combines the use of an ATM terminal owned by one member bank and the use of a bankcard (possibly) issued by another member to complete a cash withdrawal. The ATMs and bankcards are complementary products.

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However, ATMs are substitutes for one another, as are different bankcards. Credit card networks like Visa or Mastercard have a similar structure, as modelled by Baxter [1983], Phillips [1987], and Salop [1991]. So do real estate multiple listing services and other electronic and product networks. Networks vary in the way in which market competition is structured and the degree of integration among component producers.

Cournot [1838] considered the merger of two monopolists that produce complementary goods (zinc and copper) into a single (fused) monopolist that produces the combination of the two complementary goods (brass). He showed that joint ownership by a single integrated monopolist reduces the sum of the two prices, relative to the equilibrium prices of the independent monopolists. This is because the two independent firms ignore the effect of their individual markups on each other, while the integrated monopolist internalizes this externality.

We generalize the Cournot model to the case of multiple producers of differentiated brands of each component, under the assumption that components are fully compatible and the number of brands of each component is exogenous. We derive and compare the equilibrium prices under a varied set of assumptions regarding market structure. Following Cournot, we compare independent and joint ownership (i.e. full integration) of component producers. We also analyze a number of other market structures involving partial integration that may characterize various networks. Thus, we provide a basic model in which a variety of networking and product compatibility issues can be easily analyzed.

The paper is organized as follows. Section II briefly reviews the Cournot complements model. In Section III, we generalize the model to an exogenous number of multiple producers of each component. In section IV we analyze the two market structures considered by Cournot—*independent ownership*, that is, oligopoly among independently-owned price-setting component producers; and *joint ownership*, that is, full integration by all component producers into a single jointly owned monopolist. For example, independent ownership characterizes much computer hardware and software. Some firms produce hardware, while different firms produce software. Each producer sets price independently.

Joint ownership characterizes some networks. For example, Western Union sets the total price for its money transfer service as well as the division of this price between the originating and terminating agent. Similarly, Nintendo sets the price of its hardware and the price of its software. It indirectly controls the price of licensees' software by its license fee, and its control over cartridge manufacturing.

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1 Free entry and optimal variety issues are not analyzed.
Our analysis highlights the tradeoff between the welfare gains from "vertical" integration and the losses from "horizontal" integration. Joint ownership internalizes two externalities: the "vertical" externality among complements identified by Cournot, and the "horizontal" externality among competing products. Thus, prices may rise or fall according to the relationship between the own and cross elasticities of demand for the composite products. If the own elasticity is large relative to the cross elasticities, then joint ownership reduces prices, and vice versa.

In section V we consider two benchmark cases. These cases are not realistic or attainable in competitive markets, but they provide useful benchmarks for understanding the basic economic forces. First we briefly describe the first best optimum. This structure can result from perfect price regulation (i.e. price equal to marginal cost) of the markets for both complementary components. We also consider a market structure in which each composite good is sold by a different independent firm. This composite goods competition benchmark involves competition among \( N \) composite good packagers, instead of competition among \( N \) component producers. In this structure, different packagers sell differentiated complete computer systems, rather than the consumers building their own systems from components they purchase. This reference structure results in lower prices than any of the non-regulated structures analyzed because it fully internalizes all the vertical externalities while maintaining full horizontal competition.

In Sections VI and VII, we analyze alternative forms of partial integration. Parallel vertical integration perhaps is the most common network market structure. This structure has joint ownership of pairs of complementary components, along with continued competition among substitute compatible components. For example, a consumer might purchase hardware and software from the same firm or from different firms. An airline traveler on a one-stop itinerary may use the same airline ("on line") or change airlines ("interline") for the second leg of the trip. In this market structure, the gains from the (vertical) integration of complementary components are partially achieved, while competition among substitute products is maintained. Parallel vertical integration is the appropriate generalization of vertical integration in Cournot's duopoly of complementary goods. It leads to lower prices than does independent ownership.

Section VII analyzes the case of one-sided joint price setting. In this market structure, the price of one component is jointly set at marginal cost (or it might be set by government regulation), and there is independent competition among producers of the other component. For example, an ATM network might set the price (called the interchange fee) received by the ATM owner while card issuers continue to set transaction fees independently (Baxter [1983], Gilbert [1991], Kauper [1988], Salop [1990]). A multiple listing services (MLS) might set the commission to the "selling" agent while permitting price competition among "listing" agents. This structure may be
implemented in practice by requiring one component to be sold at a specified “wholesale” price to the producers of the other component, who then compete at retail by constructing and selling composite products. If the price of the component is set at marginal cost, one-sided joint price setting always results in lower prices for the composite goods than either parallel vertical integration or independent ownership. However, one-sided joint price setting leads to a higher equilibrium price than either composite goods competition or optimal price setting. We also identify which component to choose (from a social welfare perspective) to jointly set the price of (or regulate) if only one price can be jointly set. Lower prices are achieved when the less competitive of the two markets is subject to joint price setting, and we characterize the measure of competitiveness. The conclusion makes some suggestions for further work.

II. COURNOT'S MODEL OF COMPLEMENTARY GOODS DUOPOLY

Cournot's [1838] model of complementary duopoly provides a simple introduction to complementary products. Firms $A$ and $B$ are monopolistic producers of components $A$ and $B$ respectively. Marginal costs are zero, and the firms sell these components at prices $p$ and $q$ respectively. Consumers combine these two components in fixed proportions (e.g. one unit of each) to form a composite product $AB$. Demand for the composite product is denoted by $D(s)$ and depends on the sum of the two component prices, $s = p + q$. Each firm chooses price to maximize profits, taking the price of the complementary component as given. Thus, in modern terminology, we solve for the non-cooperative equilibrium (i.e. the Nash equilibrium in prices). Sonnenschein [1968] noted that this problem is the dual of the standard Cournot problem of quantity-setting firms that produce substitutes.

Under independent ownership, the two firms choose prices to maximize profits, given by,

$\Pi_A = pD(s) = pD(p+q)$  
$\Pi_B = qD(s) = qD(p+q)$

Differentiating with respect to the own price and noting that $s = p + q$, we have the two first order conditions,

$\frac{\partial \Pi_A}{\partial p} = pD'(s) + D(s) = 0$  
$\frac{\partial \Pi_B}{\partial q} = qD'(s) + D(s) = 0$

These two equations define best-response functions $p = R_A(q)$ and $q = R_B(p)$ for the two components. These can be solved for the Nash equilibrium. Summing equations (3) and (4) to define the equilibrium price $s^f$ of the composite good $AB$, we have

$s^fD'(s^f) + 2D(s^f) = 0$
Joint ownership of the two components (i.e. vertical integration) involves maximization of joint profits, $\Pi(s)$, where

$$\Pi(s) = sD(s)$$

Differentiating (6) with respect to $s$, we have

$$\partial \Pi / \partial s = s^i D'(s^i) + D(s^i) = 0$$

Comparing equations (5) and (7), we find that the price for the composite good is lower under joint ownership rather than independent ownership, or $s^j > s^i$. Thus, we have the now standard results that joint ownership or integration by complementary products firms raises welfare ([Allen [1938]]). Of course, it should be emphasized that this is a second-best result. The joint ownership price exceeds the optimal (marginal cost) price $s^0 = 0$ that would be determined by optimal regulation.

III. THE BASIC MODEL

Following Matutes and Regibeau ([1988]), and Economides ([1989a, c, 1991a]), suppose there are multiple differentiated brands of each of two components $A$ and $B$. Formally, suppose there are $m$ differentiated brands of component $A$, where brand $A_i$ has price $p_i$, $i = 1, 2, \ldots, m$. Similarly, suppose there are $n$ differentiated brands of component $B$, where brand $B_j$ has price $q_j$, $j = 1, 2, \ldots, n$. We take the number of brands as exogenous. We assume zero marginal costs for all components. We also assume full compatibility among components. Thus, brands of each component may be combined to form all $m \times n$ composite products, such as $A_iB_j$ available at total prices $s_{ij} = p_i + q_j$. The various composite goods are substitutes for one another, and demand $D^{ij}$ for composite good $A_iB_j$ depends on the vector of total prices $s$, where $s = \{s_{ij}, i = 1, \ldots, m, j = 1, \ldots, n\}$.

We confine our attention to the case of two components of each kind, $m = n = 2$. As noted later, many of the results hold for $m = n > 2$. Because the various composite goods are substitutes for one another, demand for $A_1B_1$ is

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2 Brands of the same component are substitutes among themselves, while brands of different components are complements.

3 Of course, the results are identical for positive constant marginal costs with “prices” reinterpreted as differences between prices and marginal costs.

4 As a general matter, compatibility depends on technical feasibility, along with the technological and contractual decisions of component producers. In the related literature on compatibility it has been established that if the demand for a hybrid composite good (e.g. $A_1B_2$) is as large as the demand for single-producer goods, then independent vertically integrated firms (parallel vertical integration, in our jargon), will choose full compatibility of their components (Matutes and Regibeau [1988], Economides [1989a, c, 1991a]). We assume that the demand for hybrids is of the same size as the demand for single-producer composites, and, therefore, there are strong incentives for full compatibility. Under other organizational structures that we will consider in this paper, such as independent ownership and joint ownership, provided that the number of brands is exogenous, firms have even stronger incentives to avoid incompatibilities.
decreasing in its own price, $s_{11}$, and increasing in the prices of the three substitute composite goods, $s_{12}$, $s_{21}$, and $s_{22}$. Denoting by $D_k^{ij}$ the derivative of the demand for product $A_iB_j$ with respect to the $k$th argument, $D_1^{11} < 0$ and $D_{k1}^{11} > 0$, $k \neq 1$. We derive the demand functions for the components from the demand functions for the composite goods. Since component $A_i$ is sold as a part of composite goods $A_iB_1$ and $A_iB_2$, the demand for component $A_i$ is given by

$$D^{A_i} = D^{11} + D^{12}$$

We further assume that the demand system is symmetric. In this case, the demand system can be represented by a single demand function, $D(\cdot)$, that is,

$$D^{11}(s) = D^{12}(s) = D^{21}(s) = D^{22}(s) = D(s)$$

This implies, in particular, that when the prices for all four composite goods are equal, the demand for each of them is the same.

We assume that composite goods are gross substitutes. Therefore, an equal increase in the prices of all composite goods reduces the demand of each composite good, or

$$\sum_{k=1}^{4} D_k^{ij} < 0$$

To illustrate our results for different market structures, we will analyze the case of linear demand, or

$$(8) \quad D^{11}(s_{11}, s_{12}, s_{21}, s_{22}) = a - b \cdot s_{11} + c \cdot s_{12} + d \cdot s_{21} + e \cdot s_{22}$$

where $a, b, c, d, e > 0$ and $b > c + d + e$ because the products are gross substitutes.  

IV. EQUILIBRIUM PRICING

In this section, we analyze the two basic market structures considered by Cournot, independent ownership and joint ownership. We assume that a firm

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$^5$ We follow the notational convention of reserving the first argument of the demand functions for the own price, the second for the price of the composite good that differs in the $B$ component, the third for the price of the composite good that differs in the $A$ component, and the fourth for the composite good that differs in both components. Because the arguments are arranged in this way, the signs of the partial derivatives for each argument are identical for each composite good.

$^6$ It follows immediately that an equal increase in the prices of all components implies a decrease in the demand for each component. For example, for the demand for component $A_1$, the effects of such price changes are

$$\frac{\partial D^{A_i}}{\partial p_1} + \frac{\partial D^{A_j}}{\partial p_2} + \frac{\partial D^{A_i}}{\partial q_1} + \frac{\partial D^{A_j}}{\partial q_2} = \sum_{k=1}^{4} D_k^{11} + \sum_{k=1}^{4} D_k^{12} < 0$$

$^7$ The demand functions for the other systems can be similarly written. For example,

$$D^{12}(s_{12}, s_{11}, s_{22}, s_{21}) = a - b \cdot s_{12} + c \cdot s_{11} + d \cdot s_{22} + e \cdot s_{21}$$
does not price discriminate according to whether customers purchase both components from it (e.g. who purchase the firm’s own composite product). Although sometimes such discrimination may be possible (see Whinston [1990], Matutes and Regibeau [1992], Economides [1991b]), the analysis of such price discrimination is beyond the scope of this paper.

IV(i). Independent Ownership (I)

Suppose that all component brands $A_i$ and $B_j$ are independently owned, as illustrated in Figure 1.\(^8\)

![Figure 1](image)

The profit functions of the four firms are given by,

\[ \Pi_{A_1} = p_1 D^{A_1} = p_1 (D^{11} + D^{12}) \quad \Pi_{A_2} = p_2 D^{A_2} = p_2 (D^{21} + D^{22}) \]

\[ \Pi_{B_1} = q_1 D^{B_1} = q_1 (D^{11} + D^{21}) \quad \Pi_{B_2} = q_2 D^{B_2} = q_2 (D^{12} + D^{22}) \]

Profit maximization by firm $A_1$, for example, is characterized by

\[ (9) \quad \frac{\partial \Pi_{A_1}}{\partial p_1} (p_1, p_2, q_1, q_2) = D^{11} + D^{12} + p_1 (D_1^{11} + D_2^{11} + D_1^{12} + D_2^{12}) = 0 \]

\(^8\)We denote ownership patterns by enclosing in a box the components produced by the same firm.
For the linear case we have

\[
\frac{\partial \Pi_{A_1}}{\partial p_1} = D^{11} + D^{12} + 2(-b + c)p_1 = 2a + 4(-b + c)p_1 \\
+ (-b + c + d + e)q_1 + 2(d + e)p_2 \\
+ (-b + c + d + e)q_2 = 0
\]

The solution of the four first order conditions like (9) defines the equilibrium prices \((p^*_1, p^*_2, q^*_1, q^*_2)\) under independent ownership.\(^9\) Solving the first order conditions we have the equilibrium prices under independent ownership,

\[
\begin{align*}
p^* &= a(b - d)/F \\
q^* &= a(b - c)/F \\
s^* &= a(2b - c - d)/F
\end{align*}
\]

where

\[
F = (b - c)(b - d) + (2b - c - d)(b - c - d - e) > 0
\]

IV(ii). Joint Ownership (J)

We now consider the market structure of joint ownership, or full integration, of all four component producers. We assume that in joint ownership a single decision maker maximizes joint profits. Compared to independent ownership, joint ownership creates some downward pressure on prices because of the “vertical” integration of complementary complements \((A_i \text{ merging with } B_j)\) and some upward pressure on prices because of the “horizontal” integration of substitutes (e.g. \(A_i \text{ merging with } A_j\)).

The jointly owned firm or network maximizes the sum of the profits of the four component producers, or

\[
\Pi = \Pi_{A_1} + \Pi_{B_1} + \Pi_{A_2} + \Pi_{B_2}
\]

Differentiating, we have

\[
\begin{align*}
d\Pi / dp_1 &= D^{11} + D^{12} + p_1(D_1^{11} + D_2^{11} + D_1^{12} + D_2^{12}) \\
&\quad + q_1(D_1^{11} + D_2^{11} + D_3^{21} + D_4^{21}) \\
&\quad + p_2(D_3^{21} + D_4^{21} + D_3^{22} + D_4^{22}) \\
&\quad + q_2(D_1^{12} + D_2^{12} + D_3^{22} + D_4^{22}) = 0
\end{align*}
\]

For the linear case, we have,

\[
\begin{align*}
d\Pi / dp_1 &= D^{11} + D^{12} + 2(-b + c)p_1 - (b - c - d - e)q_1 + 2(d + e)p_2 \\
&\quad - (b - c - d - e)q_2 \\
&= 2a + 4(-b + c)p_1 + 2(-b + c + d + e)q_1 + 4(d + e)p_2 \\
&\quad + 2(-b + c + d + e)q_2
\end{align*}
\]

\(^9\)When the demand is not linear, the equilibrium is derived by defining two dimensional representations of the best reply functions. Economides and Salop [1991] show that this equilibrium exists under fairly general demand conditions.
Similar conditions may be derived for the other prices. The joint ownership equilibrium for a linear demand system is the solution of the system of the first order conditions (11). Because the first order conditions are not independent, we cannot determine \( p^I \) and \( q^I \) separately. Thus, we only determine the price for composite goods,

\[
s^I = p^I + q^I = a/[2(b-c-d-e)]
\]

We now compare joint ownership with independent ownership. Comparing equation (11) with equation (9), we note that the first two terms as well as the first parenthesis are identical in both equations, and equation (9) contains no other terms. These three terms represent the effect of changes of \( p_1 \) on \( \Pi_A_i \). Of the remaining terms of equation (11), the second and the fourth parenthesis represent the effects of the vertical mergers of firm \( A_1 \) with firms \( B_1 \) and \( B_2 \). They place a negative influence on prices, relative to independent ownership. However, the third parenthesis in equation (11) is positive. It is a summation of the effects on demand of increases in the price of a substitute. It represents the effects of the horizontal merger between firms \( A_1 \) and \( A_2 \) and places a positive influence on prices. Thus, the overall effect of the full merger is ambiguous, since it depends on the relative magnitudes of the second and fourth parentheses compared to the third one.

When the composite goods are very close substitutes—that is, when the cross partial price derivatives of demand among composite goods outweigh the own partial derivatives of demand—the horizontal effects of the merger dominate. Thus an increase in the price of good \( A_1 \) increases the total sales of \( A_2 \), \( B_1 \), and \( B_2 \). This is simply another way of saying that the sum of the partial price derivatives of the demand for component \( A_1 \) with respect to components \( A_2 \), \( B_1 \), and \( B_2 \) is positive. Thus, when composite goods are close substitutes, prices rise from the integration of the four independent firms,

\[
s^I = p^I + q^I > p^I + q^I = s^I
\]

Comparing equations (12) and (10), \( s^I > s^I \) if and only if

\[
(b-c-d-e)(b-c) < (b-d)(d+e)
\]

When the cross derivatives are equal (i.e. \( c = d \)), the prices of the complementary components are equal, \( p^I = q^I \), and \( s^I > s^I \) if and only if\(^{10}\)

\[
b < 3c + 2e
\]

Thus, joint ownership raises prices when the cross partial price derivatives, \( c, d, \) and \( e \), of the demand for composite products are high, relative to the absolute value of the own price derivative of the demand, \( b \). This is because these own and cross partials define the degree of competition among

\(^{10}\) Recall that since the assumption of gross substitutes implies that \( b > 2c+e \), equation (15) defines the range of the parameters where joint ownership raises prices.
substitute components under independent ownership that is eliminated by joint ownership relative to the size of the “vertical” externality among complementary components that joint ownership internalizes. Joint ownership raises prices when there would otherwise be intense competition under independent ownership—that is, when composite goods, and different brands of the same component, are close substitutes for one another.

Proposition 1. Prices are higher in joint ownership than in independent ownership if and only if the composite goods are close substitutes.

V. BENCHMARKS

In this section we analyze two reference market structures that can serve as benchmarks, optimal regulation and composite goods competition. These structures may not be attainable in reality, but they are useful references for understanding the basic economic forces at work.

V(i). Optimal Regulation (O)

Consider the first best outcome, where a regulator imposes marginal cost pricing on all components.\footnote{This result obviously depends on the assumption of an exogenous number of brands. If the number of brands is determined by free entry in equilibrium (and entry involves, for example, a fixed cost and a constant marginal cost), optimal prices exceed marginal cost, in the absence of subsidies. Similarly, all the comparisons that follow depend on this assumption of a fixed number of brands. This clearly restricts the policy implications that flow directly from this model. See Spence [1976], Salop [1979, 1991], and Economides [1989b].} We denote the marginal cost price of the composite goods as $s^O = 0$. Thus, we have

$$s^O < \min (s^I, s^J)$$

V(ii). Composite Good Competition (C)

As a second benchmark, consider composite goods competition, in which each of the four composite goods is produced by a different firm: $i = 11, 12, 21, 22$. For example, consider the market for vacations, where the vacation composite good package is comprised of two components, airline transportation and resort hotel stay. Suppose there are two airlines and two hotels and marginal costs are zero. In independent ownership, airlines and hotels set prices and the consumers purchase components to create their own vacation package. In contrast, composite goods competition would involve competition among four travel agents, each with zero production costs, each of whom sells one of the four different vacation packages, as illustrated in Figure 2, and neither hotels nor air travel are sold separately. Thus, in composite goods competition, there are still four sellers, but the products sold
differ. We *assume* that the travel agents’ marginal costs equal the marginal costs of the components. Such composite goods competition is not realistic, but it provides a useful reference point.

One might expect prices to be the same under composite goods competition as under independent ownership, since there are four sellers in both structures. In fact, prices are always lower in composite goods competition. This is because that structure internalizes all the vertical externalities while maintaining horizontal competition.

![Composite Goods Competition](image)

Figure 2

The profit function of firm 1 is given by

$$\Pi^{11} = s_{11} D^{11} (s_{11}, s_{12}, s_{21}, s_{22})$$

Differentiating with respect to $s_{11}$, we have the first order condition,

$$D^{11} + s_{11} D_{s_{11}}^{11} = 0$$

(16)

For linear demand, the equilibrium price is given by

$$s^C = a/(2b - c - d - e)$$

(17)  

From direct comparison of (10), (12) and (17), it follows that the equilibrium prices for composite goods are lower in composite goods competition than both in independent ownership and in joint ownership—that is,$^{12}$

$$s^C < \min (s^I, s^J)$$

$^{12}$This is also true for general demand functions. For details, see Economides and Salop [1991].
VI. PARTIAL INTEGRATION

There are a variety of market structures in which there is partial integration. We analyze two important ones in this section and the next one.

VI(i). **Parallel Vertical Integration, (V)**

The case of joint ownership does not reflect Cournot's [1838] result that prices necessarily fall because of integration. Joint ownership involves both vertical and horizontal effects. The parallel vertical integration structure separates these effects. Parallel vertical integration involves the integration of compatible complementary components while maintaining competition among substitute components.

Formally, suppose that $A_i$ and $B_i$ integrate to form firm $i$, $i = 1, 2$. Firm-$i$ continues to sell its compatible components separately, however, as well as composite product $A_iB_j$. We assume no price discrimination in favor of consumers who purchase both components from the same firm. Thus, consumers can still purchase components from different firms to produce hybrid composites like $A_iB_j$ at no extra cost. Figure 3 illustrates the ownership structure of parallel vertical integration (V).

Parallel vertical integration is common in networks. Many firms produce and sell compatible complementary components in addition to a composite

![Parallel Vertical Integration](image)

Figure 3
product composed of its components. For example, in ATM networks, a consumer can obtain a cash withdrawal from an ATM at its own bank or from an ATM owned by another bank. A PC user can mix hardware and software of different companies or choose both hardware and software made by the same producer. In airline travel, both on-line and inter-line one-stop trips are often possible. Formally, the profit function for firm 1 is given by

$$\Pi^1 = \Pi_{A_1} + \Pi_{B_1} = p_1(D^{11} + D^{12}) + q_1(D^{11} + D^{21})$$

Maximizing with respect to \( p_1 \), we have\textsuperscript{13}

\begin{equation}
\frac{d\Pi^1}{p_1} = D^{11} + D^{12} + p_1(D^{11}_1 + D^{11}_2 + D^{12}_2 + D^{12}_1) + q_1(D^{11}_1 + D^{21}_2 + D^{21}_3 + D^{21}_4)
= D^{11} + D^{12} + 2(-b+c)p_1 + (-b+c+d+e)q_1 = 0
\end{equation}

The system of equations (18) is solved as follows:

(19a) \[ p^V = 2a(b+c-d+e)/F' \]

(19b) \[ q^V = 2a(b-c+d+e)/F' \]

(19c) \[ s^V = 4a(b+e)/F' \]

where \( F' = 4(2b-2c-d-e)(2b-2d-c-e)-9(b-c-d-e)^2 > 0 \).

To compare the prices in parallel vertical integration, \( s^V \), with independent ownership, \( s^I \), we compare equations (18) with (9). It is easy to show that

\[ s^V < s^I \]

**Proposition 2.** Prices are always lower in parallel vertical integration than in independent ownership.\textsuperscript{14}

The price comparison between parallel vertical integration and joint ownership is ambiguous. This is because the parallel vertical integration does not eliminate all the vertical externalities. In particular, parallel vertical integration leaves uninternalized externalities between the prices of components \( A_1 \) and \( B_2 \), and components \( A_2 \) and \( B_1 \). Thus full integration of the two pair-wise integrated firms into joint ownership may lower prices.

This integration has both vertical and horizontal effects. Integration of \( (A_1 + B_1) \) with \( A_2 \) has a positive influence on the price of \( A_1 \), while integration of \( (A_1 + B_1) \) with \( B_2 \) has a negative influence on the price of \( A_1 \). Therefore, the comparison is ambiguous, depending on the relative magnitudes of the own and cross partials.

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\textsuperscript{13} The analysis with respect to \( q_1 \) is similar.

\textsuperscript{14} Using the same technique, it can be shown that this result is true for \( m = n > 2 \). This result also holds for more general demand and profit functions. The sufficient condition is that the marginal profits of firm \( A_1 \) are decreasing with equal increases in prices \( p_1 \) and \( p_2 \). See Economides and Salop [1991].
If the composite goods are close substitutes, the cross partials term will dominate the own price derivative of demand. Thus, prices will rise as a result of the horizontal merger of the pair-wise vertically integrated firms. Alternatively, if the four composite goods are not close substitutes, the own price derivative will dominate and prices will fall as a result of the merger into joint ownership. This is seen more clearly in the case when \( c = d \). For this case, \( p^V = q^V \), and price will rise as a result of the merger into joint ownership if and only if

\[
(20) \quad b < 4c + 3e
\]

**Proposition 3.** Prices are higher in joint ownership than in parallel vertical integration if and only if the composite goods are close substitutes.

Summarizing Propositions 1 and 3, when composite goods are not close substitutes, joint ownership results in lower prices than parallel vertical integration. When goods are moderately close substitutes, full integration results in a higher price than parallel vertical integration but a lower price than independent ownership. For very close substitutes, full integration results in a price even higher than independent ownership.\(^{15}\)

The comparison between composite goods competition and parallel vertical integration is straightforward. Comparing (17) and (19c) we have

**Proposition 4.** Prices are higher in parallel vertical integration than in composite goods competition, \( s^C < s^V \).

Intuitively, composite goods competition internalizes all the vertical externalities but none of the horizontal externalities. Thus, the maximum degree of competition results.

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\(^{15}\) From (15) and (20), we have the following price comparisons for \( c = d \):

\[
\begin{align*}
\text{s}^l > \text{s}^f & \iff b < 3c + 2e \\
\text{s}^l > \text{s}^V & \iff b < 4c + 3e
\end{align*}
\]

Also note that from the assumption of gross substitutes, \( b > 2c + e \). Thus when the composite goods are not close substitutes, \( b > 4c + 3e \), full integration results in lower prices for composite goods than both independent ownership and parallel vertical integration. When the composite goods are moderately close substitutes, i.e., when \( 3c + 2e < b < 4c + 3e \), prices increase as a result of a merger of two pair-wise integrated firms but not by the merger of four independent firms. When the composite goods are very close substitutes, \( 2c + e < b < 3c + 2e \), both mergers increase prices.
"wholesale" price set jointly by the network. These latter producers then package the components into composite goods and sell them at retail to consumers. We denote this structure as one-sided joint price setting. This structure can also be interpreted as one-sided regulation, where the price of one of the two complementary components is set by a regulator. For example, natural gas producers sell natural gas and transportation on pipelines they own. Under one-sided regulation, the Federal Energy Regulatory Commission regulates the price of interstate transportation while it permits competition in the sale of natural gas.

In our simple model, the controlling component producers have an incentive to set the price of the "regulated" component at marginal cost. In this way, the regulated component producers obtain no surplus, and the vertical externality is internalized. Of course, the other components (and the composite goods) continue to sell above marginal cost. Further, we derive the optimal structure for one-sided regulation. That is, assuming that it is feasible to set only one component's price jointly, we show which would yield higher welfare.

VII(i). Equilibrium with One-Sided Joint Price Setting of Component A

Formally, one-sided joint price setting of component A sets the prices of A-brands at marginal cost, or \( p_i = 0, \ i = 1, 2 \). At the same time, the prices of brands of component B are set independently and non-cooperatively. The solution of the system

\[
\frac{\partial \Pi_{A_i}}{\partial p_i} = 0, \ i = 1, 2,
\]
evaluated at \( q_j = 0, j = 1, 2 \), is

\[
(21) \quad s^R_A = p^R_A = a/(2b - 2c - d - e)
\]

This is a lower price than the price under independent ownership, \( s^I \). Therefore we have,

Proposition 5. Composite goods prices are lower under one-sided joint price setting (regulation) than under independent ownership, \( s^R < s^I \).17

This result that the prices of composite goods fall is not surprising. By setting the price of one component at marginal cost while maintaining duopoly competition among producers of the other component, the negative

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16 Price would fall further if both components were priced at marginal cost, so that regulation of both components would be optimal.

17 This result easily generalizes to the general case of linear demand with \( m, n > 2 \) firms. This result also holds for more general demand and profit functions. The sufficient condition is that the marginal profits of firm \( A_1 \) are decreasing with equal increases in prices \( p_1 \) and \( p_2 \). See Economides and Salop [1991].
vertical externality is eliminated, relative to independent ownership. It also follows immediately that the producers of the other component gain from the price “regulation” of the other side of the market. This also implies that producers of one component have the incentive to engage in collective action to force down the price of the other component, even if they continue to compete among themselves.\textsuperscript{18}

Compared with parallel vertical integration, one-sided joint price setting results in lower prices for composite goods. One-sided joint price setting eliminates completely the market power of the producers of one of the components, while the vertically integrated firms have some market power in the market for each component.

**Proposition 6.** Composite goods prices are lower in the regime of one-sided joint price setting (regulation) than under parallel vertical integration, $s^R < s^V$.\textsuperscript{19}

To summarize, we have shown that the prices of systems in the one-sided joint price setting are lower than in parallel vertical integration and independent ownership, but higher than in the hypothetical market structures of composite goods competition and optimal regulation, i.e.

$$s^O < s^C < s^R < s^V < s^I$$

It is also easy to show using the same reasoning as in Proposition 6, that joint ownership results in a higher price than one-sided joint price setting, or

$$s^R < s^J$$

VII(ii). *Optimal One-sided Joint Price Setting*

One-sided joint price setting results in a lower price than independent ownership no matter which component is chosen. This raises the issue of

\textsuperscript{18} The equilibrium prices for systems arising in the one-sided joint price setting structure also obtain in a market structure involving partial integration of composite goods competitors, as follows. Recall that the four producers in the composite goods competition structure sell the four goods $A_1B_1$, $A_1B_2$, $A_2B_1$, and $A_2B_2$ respectively. Suppose that the two firms that use component $A_1$ (i.e. $A_1B_1$ and $A_1B_2$) integrate into a firm that we denote as $IA_1$. Similarly, suppose that the two firms that use component $A_2$ (i.e. $A_2B_1$ and $A_2B_2$) integrate into a firm that we denote as $IA_2$. Suppose further that these two integrated firms, $IA_1$ and $IA_2$, continue to compete non-cooperatively. We denote this market structure as *partially integrated composite goods competition*. It is shown in Economides and Salop [1991] that this structure is formally equivalent to and results in the same composite goods prices as one-sided joint price setting, where the price of the $B$-type goods is set to marginal cost. Since this structure can arise as the result of horizontal mergers from composite goods competition, it results in a higher price, $s^C < s^R$.

\textsuperscript{19} This result generalizes to the same extent and under the same sufficient conditions as in footnote 17.
which side to choose if only one component is subject to joint price setting (or, say, only one component can be subject to formal regulation).

Intuition suggests that jointly setting a low price in the less competitive market will result in the lower price for the composite good. The less competitive market is characterized by a smaller number of components and/or smaller cross price partials of demand between the components. When the less competitive price is set equal to marginal cost, a larger price decrease is achieved. Then, continued competition among the brands of the unregulated component will maintain a low price in that market. Thus, it is better to regulate the $A$ side of the market if and only if $c > d$.\textsuperscript{20}

VIII. CONCLUSION

We have analyzed competition and integration among complementary products in networks by examining a variety of alternative market structures. We have shown that different market structures internalize “vertical” and “horizontal” externalities in various ways. For example, in the hypothetical market structure of composite goods competition, externalities among complementary components are fully internalized while maintaining competition among substitute systems.

We have also shown that parallel pair-wise vertical integration generalizes Cournot’s [1838] result that mergers among complements reduce prices. However, we noted that a merger of all firms in the industry may or may not increase prices, depending on the relative sizes of the own and cross partials of demand. We also showed that one-sided joint price setting, which in effect limits monopoly power to one side of the market only, results in lower prices than independent ownership, and we have characterized which component should be chosen for joint price setting.

The analysis in this paper can be extended and generalized in a variety of ways. First, many of our results pertain to the case of only two brands of each component. We believe that they can be extended to the case where each component has many brands. Some of our results assumed linear demand. Thus, our analysis can be extended to a broader range of cases. Second, we assumed that the number of brands is exogenous. This limits the generality of the model as well as its applicability to some network policy issues. When the number of brands is endogenous, the analysis is complicated by issues of product variety. As a result, the welfare implications of price comparisons are less clear and optimal network self-regulation is far more complicated. Third, we assumed that integrated firms do not price discriminate in favor of

\textsuperscript{20} For a proof, see Economides and Salop [1991]. In the general model with $(m, n)$ firms, it is better to regulate the $A$ side of the market if and only if $(n-1)c > (m-1)d$. Thus, a market is “more competitive” if there are more competing firms, and the components are closer substitutes.
customers who purchase both components (i.e. who purchase the firm’s own system). By relaxing these assumptions, a richer set of strategies that may be important in certain product networks can be analyzed.

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